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Neutrinos as cluster dark matter

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ABSTRACT

The dynamical mass of clusters of galaxies, calculated in terms of MODified Newtonian Dynamics (MOND), is a factor of 2 or 3 times smaller than the Newtonian dynamical mass but remains significantly larger than the observed baryonic mass in the form of hot gas and stars in galaxies. Here I consider further the suggestion that the undetected matter might be in the form of cosmological neutrinos with mass of the order of 2 eV. If the neutrinos and baryons have comparable velocity dispersions and if the two components maintain their cosmological density ratio, then the electron density in the cores of clusters should be proportional to $T^{3/2}$, as appears to be true in non-cooling flow clusters. This is equivalent to the ‘entropy floor’ proposed to explain the steepness of the observed luminosity–temperature relation, but here preheating of the medium is not required. Two-fluid (neutrino–baryon) hydrostatic models of clusters, in the context of MOND, reproduce the observed luminosity–temperature relation of clusters. If the β law is imposed on the gas density distribution, then the self-consistent models predict the general form of the observed temperature profile in both cooling and non-cooling flow clusters.

Key words: gravitation – galaxies: clusters: general – intergalactic medium – dark matter – X-rays: general.

1 INTRODUCTION

In the past 25 yr, Milgrom’s proposed alternative to dark matter, the MODified Newtonian Dynamics or MOND (Milgrom 1983), has enjoyed considerable phenomenological success with respect to galaxy scaling relations and individual rotation curves (see Sanders & McGaugh 2002, for a review). The range of this predictive power has recently been extended from very low-luminosity objects (Milgrom & Sanders 2007) to luminous early-type spirals (Sanders & Noordermeer 2007). However, for some time it has been recognized that MOND does not fully account for the mass discrepancy in rich clusters of galaxies (The & White 1988; Gerbal et al. 1992; Sanders 1999; Aguirre, Schaye & Quataert 2001; Sanders 2003; Pontecouteau & Silk 2005). This is basically because, with MOND, the departure from Newtonian dynamics appears below a critical acceleration, a_0 . The value of a_0 determined from galaxy rotation curves is about $10^{-8} \text{ cm s}^{-2}$. In clusters of galaxies, the observed acceleration (estimated from the hydrostatic gas equation) is typically greater than a_0 in the central regions, so MOND cannot alleviate the observed discrepancy there. When the MOND version of the hydrostatic gas equation is applied to the observed temperature and density distributions in X-ray emitting clusters of galaxies, one finds that there remains a discrepancy roughly of a factor of 3 between the identifiable baryonic mass in gas and stars and the dynamical mass.

That is to say, MOND also requires the presence of undetected, or dark, matter in clusters of galaxies, albeit significantly less than conventional Newtonian dynamics.

X-ray and weak-lensing observations of the famous ‘bullet cluster’ (Clowe et al. 2006) are now presented as definitive evidence for non-dissipative dark matter in clusters of galaxies and, by extension, as evidence against MOND. Had it been previously claimed that MOND fully accounts for the kinematic and X-ray observations of clusters without dark matter, then this object would indeed have been quite problematic for the theory. But, in fact, the bullet creates no *additional* difficulties for MOND; the quantity of dark matter required is consistent with that suggested by the previous analyses (Angus et al. 2007). What the bullet cluster adds (and this is a significant addition) is convincing evidence that the dark component cannot be dissipative like the extended X-ray emitting gas.

At first thought, it might appear negative for MOND that additional dark matter is required in the cluster environment. But one could also consider this to be a bold prediction – that there is more matter to be discovered in clusters. For example, there are more than enough undetected baryons to make up the missing dark component; they need only be present in some non-dissipative form which is difficult to observe. Moreover, it is now known that at least two of the three active neutrino types have non-zero mass (Fukuda et al. 1998). Primordial neutrinos are present in the Universe with a number density comparable to that of photons, so non-baryonic dark matter certainly exists. If the neutrino mass scale is as large as 2 eV, then neutrinos would comprise a non-baryonic component of

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rich clusters (Sanders 2003), but because of phase-space constraints (Tremaine & Gunn 1979) they could not accumulate in individual galaxies. β decay experiments have restricted the mass of the electron neutrino to be less than about 2.2 eV, so it remains possible that this is the appropriate mass scale (Groom et al. 2000). The addition of cosmological neutrinos with mass of this order produces a reasonable fit to the observed angular power spectrum of the cosmic microwave background (CMB) radiation, through the second peak, without the presence of cold dark matter (CDM; McGaugh 2004). Such a component also appears to be necessary to match the observed CMB fluctuations in the context of the recently proposed covariant theory of MOND (Bekenstein 2004; Skordis et al. 2006).

Therefore, here I consider further the possibility that the missing component in clusters of galaxies in the context of MOND may be neutrinos. I demonstrate that the form of correlations between observable quantities – the size–temperature relation ($R \propto T$) and the gas mass–temperature relation ($M_g \propto T^2$) are consistent with MOND expectations without any additional astrophysical mechanism other than violent relaxation. However, the luminosity–temperature relation ($L \propto T^3$) is not consistent with MOND by itself; an additional dark component seems to be required (Sanders 2003).

For a sample of well-studied clusters, the MOND dynamical mass is proportional to but a factor of 3 or 4 times larger than the observable gas mass. This suggests that the detectable baryonic mass is a fixed fraction of the dark mass. If the mass of each active neutrino type is about 2 eV, then the ratio of neutrino to baryonic cosmological mass densities would be about 2.8, consistent with the inferred (via MOND) ratio of gas to dark matter in clusters.

I consider the structure of two-component isothermal objects consisting of neutrinos and gas. The neutrino fluid is described by the equation of state of partially degenerate fermions, where the degree of degeneracy in the centre is arbitrarily set to a specified value. The maximum density of neutrinos is proportional to $T^{3/2}$. This means that, in a uniformly mixed two-component fluid, the central gas density would also be proportional to $T^{3/2}$. This appears to be the case in non-cooling flow clusters where there has been no inward flow of gas resulting in a rearrangement of the cosmological density ratio of baryons to neutrinos. With such models, the observed luminosity–temperature scaling relation for clusters is recovered primarily because $L \propto n_e^2 \propto T^3$.

I then consider individual clusters with the two-fluid model. After specifying the mass scale of the neutrinos and the degree of degeneracy in the centre, I assume that the presumably constant neutrino velocity dispersion is equal to that of the gas as implied by the mean emission-weighted gas temperature. Given the observed gas density distribution (via β model fits to the X-ray intensity distribution) I calculate the radial dependence of the gas temperature which is consistent with this density distribution. The only free parameter is the central gas temperature which must lie in a narrow range in order to yield a sensible temperature distribution (one in which the temperature does not rapidly increase to large values or fall to zero). These calculated temperature distributions are found to be generally consistent with those now observed – a slowly decreasing temperature for non-cooling flow clusters, and a temperature which first increases with radius and then decreases for cooling flow clusters.

I conclude that the two-fluid neutrino–baryon model for clusters of galaxies in the context of MOND is consistent both with the correlations between observable quantities and with the observed density and temperature distributions in individual clusters.

In all that follows I have scaled observational results and correlations to $H_0 = 72 \text{ km s}^{-1} \text{ Mpc}$.

2 GLOBAL SCALING RELATIONS FOR CLUSTERS OF GALAXIES

It is of interest to consider correlations between the observed, and not the inferred, properties of X-ray emitting clusters of galaxies. One of the most obvious correlations is that between the cluster size (out to a specified X-ray intensity level) and the temperature of the hot gas. Mohr et al. (2000) find that this relation, for nearby clusters, is

$$R = 0.5 \left(\frac{T}{6 \text{ keV}} \right)^{1.02} \text{ Mpc} \quad (1)$$

with a surprisingly low scatter of about 15 per cent. With conventional dynamics, assuming self-similar collapse, the expectation is more like $R \propto T^\alpha$ where $\alpha \approx 1/2$ – $2/3$. Mohr et al. explain the difference between the observations and the expectations by noting that the fraction of gas (as opposed to the total baryonic fraction including the luminous matter in galaxies) appears to increase with cluster temperature. While this seems to be a general trend (David et al. 1990; Edge & Stewart 1991), the observed relationship between gas mass/total baryonic mass and temperature exhibits enormous scatter. It is unclear how such a tight correlation can survive.

In terms of MOND, equation (1) takes on a different meaning. The mean internal gravitational acceleration in clusters is given roughly by $a = \sigma^2/R$ (σ is the velocity dispersion and proportional to temperature). Then we may interpret equation (1) as defining a single-characteristic internal acceleration which is independent of cluster size or temperature; that is, $a \propto T/R \approx 6 \times 10^{-9} \text{ cm s}^{-2}$. This is within a factor of 2 of the MOND critical acceleration ($a_0 \approx 10^{-8} \text{ cm s}^{-2}$). It has been pointed out previously that in pressure supported, nearly isothermal systems, the internal acceleration is approximately the MOND acceleration (Sanders & McGaugh 2002). Viewed in this way, the cluster temperature–size relation simply reflects that characteristic internal acceleration and requires no astrophysical input other than the requirement of a near isothermal state, presumably due to violent relaxation.

A second correlation is that between the temperature and the observed gas mass. Mohr, Mathiesen & Evrard (1999) give

$$\frac{M_g}{10^{14} M_\odot} \approx 0.017 T_{\text{keV}}^2. \quad (2)$$

For self-similar collapse, making use of the Newtonian virial theorem, the expectation would be $M_g \propto T^{3/2}$. The disagreement between observations and expectations is again attributed to a systematically changing fraction of baryons in hot gas perhaps resulting from increasing energy injection into the intracluster medium or a decreased efficiency of galaxy formation in hotter clusters.

With MOND, however, equation (2) would be the expectation from the dynamics of isothermal (or semi-isothermal) spheres; it is, in effect, an extrapolation of the Faber–Jackson relation for elliptical galaxies (Sanders 1994) and, in the case of MOND would apply to all pressure supported, near isothermal systems (Milgrom 1984). Basically, the equation of hydrostatic equilibrium in the MOND regime yields

$$\frac{M}{10^{14} M_\odot} \approx 0.06 T_{\text{Kev}}^2 \quad (3)$$

for the total dynamical mass in terms of MOND. Here I have assumed an isothermal β model with an outer logarithmic density gradient of -3β where $\beta = 0.6$ on average. Although the form of the temperature–gas mass relation is predicted, the normalization is not; the MOND dynamical mass implied by equation (3) is roughly 3.6 times larger than the observed gas mass.

The third correlation, and the most discussed in the literature, is that between luminosity and temperature. Again the correlation is well fitted by a power law; for example, Ikebe et al. (2002) give

$$L_x = 4 \times 10^{42} T_{\text{keV}}^{2.5} \text{ erg s}^{-2} \quad (4)$$

for the total X-ray luminosity between energies of 0.1–2.4 keV. Others find exponents closer to 3 with suggestion of a steepening for cooler clusters (Arnaud & Evrard 1999). The expectation for self-similar cluster formation in the context of CDM is a shallower power law $L \propto T^2$. Here, preheating of the intergalactic medium (heating before cluster formation) by winds from forming galaxies is invoked to provide an ‘entropy floor’ for the subsequent intracluster medium (entropy is defined as $T_{\text{keV}}/n_e^{2/3}$). This would have the effect of inflating the intracluster medium particularly in lower temperature galaxies and steepening the predicted luminosity–temperature relation (Ponmon, Cannon & Navarro 1999).

MOND, in this case, fares worse. In MOND a virialized system has a characteristic density $\rho_M \propto a_0^2/(GT)$. As we saw above, the characteristic radius of the cluster is $R \propto T/a_0$. For free-free radiation, $L \propto \rho^2 T^{1/2} R^3$; therefore, the naive prediction would be $L \propto T^{3/2}$. I discussed this point previously (Sanders 2003) and noted that a dark matter component with a constant surface density and a core radius roughly twice that of the gas core radius could bring the MOND prediction in line with the observations.

3 THE MAGNITUDE OF THE DISCREPANCY WITH MOND

MOND predicts more mass than is directly observed in clusters of galaxies. Determination of the magnitude of this discrepancy, however, is not straightforward. For an isothermal gas, the hydrostatic gas equation in the MOND limit implies that the mass within radius R is given by

$$M_m = \alpha^2 \left(\frac{kT}{wm_p} \right)^2 (Ga_0)^{-1}, \quad (5)$$

where $w = 0.62$ is the mean atomic weight, m_p is the proton mass and $\alpha = d \ln \rho / d \ln r$ is the logarithmic density gradient evaluated at R . The gas density distribution is typically described by the β model used to fit the X-ray surface brightness distribution:

$$\rho = \rho_0 \left[1 + \left(\frac{r}{r_c} \right)^2 \right]^{-1.5\beta}. \quad (6)$$

Therefore beyond three or four core radii $\alpha \rightarrow -3\beta$ – that is, the predicted dynamical mass, *a la* MOND, converges. On the other hand, $\beta = 0.5$ –1.0 which means that the gas mass continues to increase with radius (the β model surely must have a limited range of viability). Therefore, the magnitude of the discrepancy (defined as M_m/M_g), as estimated from the β model, decreases with radius; that is, the discrepancy depends upon the radius within which the estimate is made.

I have determined the MOND dynamical mass and gas mass for a well-studied sample of clusters given by Reiprich & Böhringer (2002). I make use of the radius–temperature relation to estimate the cut-off radius, but I take this limiting radius to be about 20 per cent larger than that given by equation (1). This implies that the cut-off is, on average, about six core radii which is a probable range of validity for the β model fit. Since the accelerations in the central regions of the clusters are typically of the order of a_0 , the form of the MOND interpolating function plays a role; here I take the simple form

$$\mu(x) = \frac{x}{1+x}, \quad (7)$$

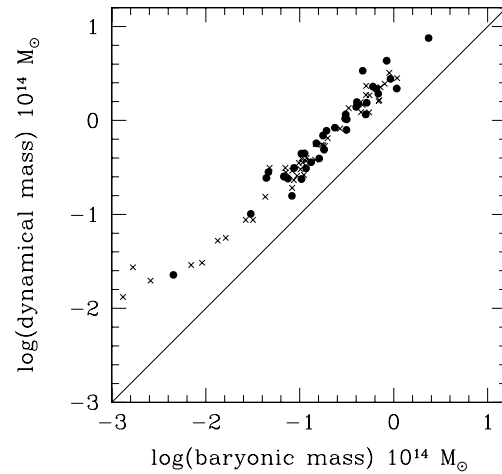


Figure 1. The logarithm of the MOND dynamical mass plotted against the logarithm of inferred baryonic mass (both in units of $10^{14} M_\odot$). The crosses indicate cooling flow clusters (those where the central cooling time-scale is less than the Hubble time), and the solid points are non-cooling flow clusters.

where $x = a/a_0$ with $a = \alpha kT/(wm_p R)$ (Famaey & Binney 2005; Zhao & Famaey 2006). The MOND dynamical mass is then given by

$$M_m = \frac{x M_N}{1+x}, \quad (8)$$

where the Newtonian mass is $M_N = -\alpha R kT/(Gwm_p)$. The gas is only a fraction of the total observable baryonic mass – the remainder being in the form of stars in galaxies. As before, I take the ratio of intracluster gas to stellar mass as given by $T_{\text{keV}}/2.5$ (Sanders 1999). There is considerable scatter about this approximate relation.

The result of this analysis is shown in Fig. 1 which is a plot of the MOND dynamical mass against the inferred baryonic mass out to a limiting radius given by $100 T_{\text{keV}}$ kpc. The ratio of the MOND dynamical mass to the baryonic mass is, on average, 3.7 with no significant difference between the cooling flow and non-cooling flow galaxies. That is to say, the observed baryonic mass in clusters accounts for one-third to one-fourth of the MOND dynamical mass.

It is evident from Fig. 1 that the ratio of dynamical to baryonic mass is roughly independent of the baryonic mass. There is an indication of a deviation at low values of the baryonic mass possibly because of an increased fraction of matter in the stellar component of galaxies, but overall, the points lie parallel to the line of equality. This is of interest because it suggests that a fixed cosmic ratio of dark to baryonic matter may be sampled by the cluster medium.

4 THE ROLE OF MASSIVE COSMOLOGICAL NEUTRINOS

If neutrinos are more massive than a few tenths of an electron volt, then, because of the small mass differences, all types have about the same mass. The cosmological density of neutrinos would then be given by

$$\Omega_\nu = 0.062 m_\nu, \quad (9)$$

where m_ν is the mass of a single neutrino type in eV. In this relation I have assumed three left-handed neutrino types and their antiparticles, but the formula would apply for Majorana as well as Dirac neutrinos.

From the standard analysis of the CMB fluctuations (Spergel et al. 2006), as well as the observed primordial abundances of helium

and deuterium (Fields & Sarkar 2006), one finds $\Omega_b = 0.044$ with $h = 0.72$ (McGaugh's analysis of CMB fluctuations without CDM implies a smaller value of 0.033 which is more consistent with the inferred primordial lithium abundance; however, given the uncertain impact of a proper relativistic theory I take the standard value). This means, from equation (9) above, that

$$\frac{\Omega_v}{\Omega_b} = 1.4 m_v. \quad (10)$$

If we suppose that clusters of galaxies sample this universal neutrino–baryon density ratio then the dynamical mass-to-baryonic mass ratio implied by Fig. 1 would mean that $m_v \approx 2$ eV.

Cosmological neutrinos freeze out of the primordial fireball below temperatures characteristic of the weak interaction scale of a few meV. The phase-space distribution is Fermi–Dirac with a maximum for each species of one particle per cell with volume $2h^3$; that is, one-half that of complete degeneracy. This initial phase-space density is maintained as a limit on the final phase-space density of any collapsed virialized object (Tremaine & Gunn 1979); of course, the degeneracy limit is absolute.

In the formation of a cluster scale object out of the mixed fluid of neutrinos and baryons, it is expected that the two fluids attain the same velocity dispersion via violent relaxation (Kull, Treumann & Böhringer 1997). With this assumption, the maximum contribution of each neutrino type (including its antiparticle) to the density of the neutrino fluid is

$$\rho_v = (2\pi)^{3/2} m_v^4 \sigma^3 h^{-3} \quad (11)$$

(Kull, Treumann & Böhringer 1996). For three neutrino types (and their antiparticles) this translates to a mass density of

$$\rho_{\max} = 1 \times 10^{-28} \left(\frac{m_v}{1 \text{ eV}} \right)^4 T_{\text{keV}}^{3/2} \text{ g cm}^{-3}. \quad (12)$$

If the ratio of the densities of baryons and neutrinos in clusters is the same as the cosmological ratio (equation 10), this means that the density of electrons, in a fully ionized plasma, would be

$$n_e = 3.5 \times 10^{-5} \left(\frac{m_v}{1 \text{ eV}} \right)^3 T_{\text{keV}}^{3/2} \text{ cm}^{-3}. \quad (13)$$

Therefore, if the ratio of baryon-to-neutrino densities in clusters is identical to the cosmic ratio, as one might expect if there has been no subsequent cooling and inflow of baryons, then the central electron density should increase as $T^{3/2}$. This is an observational prediction which can be tested, and in Fig. 2 we see the central electron density of the cluster sample of Reiprech & Böhringer plotted against the mean electron temperature. The solid points indicate those clusters where the cooling time is more than 10^{10} yr (non-cooling flow clusters) and the crosses are those clusters with cooling time-scales less than this value (cooling flow clusters). The solid line is equation (13) with $m_v = 1.9$ eV. We see that in those objects where no cooling and inflow of gas is expected – where the cosmic ratio of baryons to neutrinos is maintained – there does appear to be such a correlation.

It is interesting to note that the electron density–temperature relation imposed by the neutrino density limit and the assumption of fixed baryon-to-neutrino ratio (equation 13) would also correspond to a constant entropy, $T_{\text{keV}}/n_e^{2/3}$. Imposing such a temperature-dependent limit on the electron density is equivalent to imposing an entropy floor. This is, in fact, the very mechanism which has been suggested as to steepen the predicted luminosity–temperature relation, but here the limit is not generated by preheating but by

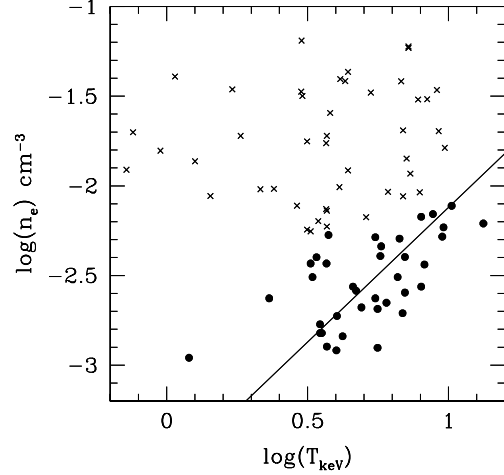


Figure 2. The logarithm of the central electron density (from the fitted β models of Reiprech & Böhringer) versus the logarithm of the temperature in eV. The points are the non-cooling flow clusters and the crosses are the cooling flow clusters. The line corresponds to equation (13) for a neutrino mass of 1.9 eV.

the phase-space constraints on the neutrino fluid. It is interesting that the entropy limit corresponding to equation (13) would be 230 keV cm^2 for $m_v = 2$ eV which is very near the value proposed to solve problem of cluster scaling relations (Lloyd-Davies, Ponman & Cannon 2000).

Before considering the implications for the luminosity–temperature relation it is necessary to describe, in a general way, the structure of a self-gravitating object consisting entirely of neutrinos near the degeneracy limit. The object can be approximated as a $\gamma = 5/3$ polytrope; the structure would be roughly that of a constant density core with a rapid decline beyond a core radius. The core radius can be estimated by combining the density temperature relation (equation 11 or 12) with an appropriate temperature–mass (or viral) relation. There are two such possible relations. The first, for Newtonian dynamics, is

$$\sigma^2 = \frac{4\pi}{10.5} G \rho_c R_c^2, \quad (14)$$

where σ is the one-dimensional velocity dispersion, and I have taken the potential energy of a $\gamma = 5/3$ polytropic sphere. The second is for MOND,

$$\sigma^4 = \frac{16\pi}{243} G \rho_c R_c^3 a_0 \quad (15)$$

(Milgrom 1998). Then with equation (12) we find two expressions for the core radius: Newtonian,

$$R_c = 4.5 \left(\frac{m_v}{1 \text{ eV}} \right)^{-2} T_{\text{keV}}^{-1/4} \text{ Mpc} \quad (16)$$

and MOND,

$$R_c^* = 1.8 \left(\frac{m_v}{1 \text{ eV}} \right)^{-4/3} T_{\text{keV}}^{1/6} \text{ Mpc}. \quad (17)$$

These two values are equal when

$$T_{\text{keV}} = 9 \left(\frac{m_v}{1 \text{ eV}} \right)^{-8/5}. \quad (18)$$

This meaning of the final expression is that when the temperature is less than about 3 keV (for $m_v \approx 2$ eV) the cluster core is in the

MOND regime and equation (17) applies; for higher temperatures the cluster core is the Newtonian regime with core radius given by equation (16). In either case, this characteristic radius is of the order of 800 kpc and depends very weakly on temperature in both regimes.

Now assuming that baryons are mixed into this cluster with the cosmic ratio, most of the X-ray emission would be coming from the core region. The X-ray luminosity is $L \propto n_e^2 T^{1/2} R_c^3$ or

$$L \propto T^4$$

for cooler clusters (MOND regime) and

$$L \propto T^3$$

for warmer clusters (Newtonian regime). In this way, the steeper than expected luminosity–temperature relation can be understood in terms of the mixed neutrino–baryon fluid. The transition from Newton to MOND would explain the steepening of the correlation for lower temperature clusters.

This, of course, is only approximate. X-ray emission also originates beyond the core, and the gas is not isothermal. In the next section I derive detailed two-fluid models of clusters.

5 MOND–NEUTRINO–BARYON MODELS OF CLUSTERS

The contribution of neutrinos to the mass budget of clusters has been considered previously in the context of mixed CDM–HDM models (Kofman, Pogosyan & Henry 1996; Treumann, Kull & Böhringer 2000; Nakajima & Morikawa 2007). In the process of collapse and virialization of a multifluid mixture of neutrinos and baryons (and/or CDM), violent relaxation (Lynden-Bell 1967) is expected to produce equal velocity dispersions for the various components, that is, $\sigma_b = \sigma_v$. I assume this condition here, although it may not be strictly true; for example, violent relaxation may be incomplete in the outer regions of clusters. Moreover, recent observations have demonstrated that the hot gas in clusters is not isothermal, in general. The implications for the hypothetical neutrino fluid are unclear, but I will assume an isothermal condition with the neutrino velocity dispersion equal to the emission-weighted velocity dispersion of the gas.

It should also be kept in mind that the density given by equation (12) is an upper limit to the density of the neutrino fluid; this value follows from the limit imposed by initial phase-space density via the collisionless Boltzmann equation (Tremaine & Gunn 1979); the course grained phase-space density in a final relaxed virialized system, and hence the space density, may be significantly less than this value. Moreover, while the neutrino fluid may be partially degenerate in the core, it will certainly depart from degeneracy in the outer regions where the space density declines and the velocity dispersion remains constant. Therefore, with respect to the neutrino fluid, we need to consider the equation of state of partially degenerate matter.

With these caveats in mind, I calculate the structure of the neutrino–baryon fluid by applying the hydrostatic gas equation separately to each fluid:

$$\frac{1}{\rho_i} \frac{dP_i}{dr} = -g, \quad (19)$$

where the subscript i refers to one of the two-fluid components, P_i is the pressure of that component, ρ_i is the density and g is the total gravitational acceleration resulting from the two components and given by the simple MOND expression,

$$g\mu\left(\frac{g}{a_0}\right) = g_N \quad (20)$$

with μ given by equation (7) and the g_N , the Newtonian force, given by $g_N = G[M_v(r) + M_b(r)]/r^2$.

The pressure of the baryonic component is

$$P_b = \rho_b \sigma_b^2. \quad (21)$$

The velocity dispersion for the baryons σ_b^2 (the temperature) may be specified as a constant (isothermal) or may vary with radius if the density distribution is specified.

The equation of state for a partially degenerate neutrino gas is given parametrically by

$$\rho_v = \frac{8\pi g_v}{\sqrt{2}} \eta \sigma_v^3 F_{1/2}(\chi) \quad (22)$$

and

$$P_v = \frac{8\pi\sqrt{2}g_v}{3} \eta \sigma_v^5 F_{3/2}(\chi), \quad (23)$$

where g_v is the statistical weight, $\eta = m_v^4/h^3$ and

$$F_p(\chi) = \int_0^\infty x^p [1 + \exp(x - \chi)]^{-1} dx \quad (24)$$

(see e.g. Landau & Lifshitz 1980). Here the degeneracy factor χ is the chemical potential divided by the temperature; large positive values of χ would correspond to complete degeneracy ($P_v \propto \rho_v^{5/3}$) and large negative values, to complete non-degeneracy ($P_v \propto \rho_v \sigma_v^2$).

The density and the velocity dispersion formally determine the chemical potential, but there is no independent method, short of numerical calculations of collapse of the two-fluid mixture to estimate χ . This essentially depends upon how close the actual central density of the neutrino fluid is to the maximum permitted by the phase-space constraints and the effectiveness of violent relaxation in equalizing the velocity dispersions of the two fluids. There are various arguments suggesting that the true density should range between 10 and 100 per cent ρ_{\max} (Madsen & Epstein 1984; Kofman et al. 1996; Nakajima & Morikawa 2007); here I take $\rho_v \approx \rho_{\max}$ because this is in rough agreement with the observational results shown in Fig. 2 (i.e. the central neutrino density is 50 per cent that corresponding to quantum mechanical degeneracy). This corresponds to $\chi = 0.32$, and in the calculations described below, I have fixed the central value of χ at this value. Realistically, due to random initial conditions for cluster formation, one would expect a dispersion in this parameter.

Assuming an identical mass ($m_v = 1.9$ eV) for all three neutrino types, a constant temperature for both fluids (with the velocity dispersion of neutrinos equal to that of the baryons), and a mass ratio of the two components equal to the cosmic density ratio (2.68 in this case), the structure of the object may be determined by numerical integration of equations (19) supplemented by equations (20)–(24). With these assumptions and constraints, the structure completely determined when the gas temperature is specified. Fig. 3 shows the density distribution for the two fluids in the case where $T_{\text{keV}} = 6$. The density of the two components effectively tracks each other in the inner regions. The distributions can be described as a roughly constant density core extending to about 300 kpc, followed by a rapid decrease (asymptotically $\rho \propto r^{-3.5}$) for both components.

The X-ray luminosity resulting from free–free emission of the hot ionized gas may also be calculated for any such object. The resulting luminosity–temperature relation is shown in Fig. 4 compared to the sample of Ikebe et al. (2002). This is similar to the figure shown in Sanders (2003) but there the dark component was added arbitrarily, with no underlying physics. Here the X-ray emission is that from self-consistent neutrino–baryon fluid spheres with only

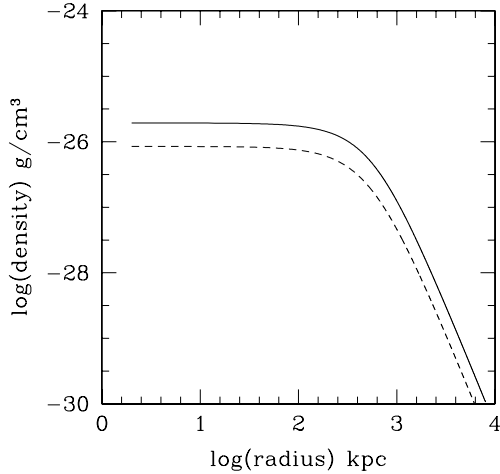


Figure 3. The logarithm of the neutrino fluid density with $m_\nu = 1.9$ eV (solid curve) and the baryon fluid density (dashed curve) as a function of the logarithm of radius. The two fluids are assumed to be isothermal corresponding to the gas temperature of 5 keV and to have a mass ratio equal to the cosmic density ratio (2.68 in this case). The central degeneracy factor is $\chi = 0.32$ implying that the central neutrino density is comparable to the maximum density imposed by the initial phase-space constraint (equation 12). The accelerations near the core radius (≈ 200) kpc are comparable to a_0 .

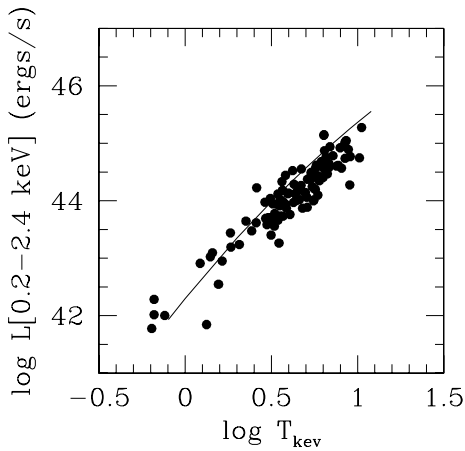


Figure 4. A log-log plot of the luminosity–temperature relation for self-consistent isothermal MOND–neutrino–baryon fluid spheres compared to the observations of Ikebe et al. (2002). The neutrino mass is taken to be 1.9 eV and the central degeneracy factor is 0.32. With the assumptions of equal velocity dispersions for the two fluids and cosmic ratio of baryons-to-neutrinos the structure and X-ray luminosity of these objects is completely specified.

the neutrino mass and degeneracy factor specified arbitrarily. The results generally agree with the observed relation; in particular the steepening of the relation for low-temperature clusters is evident.

6 NON-ISOTHERMAL β MODELS

The gas density distribution in real clusters does not resemble that shown in Fig. 3. The density in these isothermal models falls too rapidly beyond the core to be consistent with the observed distribution of X-ray surface brightness in clusters; that is, the power law implied by the fitted β models (equation 6) is more like r^{-2} rather than $r^{-3.5}$. However, it is now known that the gas in clusters

cannot be described as isothermal. There appears to be a characteristic temperature profile in clusters: in non-cooling flow clusters the temperature declines by 20 or 30 per cent between 100 kpc and 1 Mpc; in cooling flow clusters there is first a rapid rise in the inner core region (by as much as a factor of 2) followed by the decline observed in non-cooling flow clusters De Grandi & Molendi (2002); Piffaretti et al. (2005); Vikhlinin et al. (2005); Pratt et al. (2006).

Therefore, here I take a different approach. I assume the density distribution of the gas is given by a β model and I solve equations (19) and (20) to determine the temperature distribution for the gas. The neutrino fluid is again assumed to be isothermal with a velocity dispersion equal to that implied by mean emission-weighted temperature of the gas. In this way, one may predict the temperature distribution in a particular cluster with a fitted β model. The only adjustable parameter is the central value of the gas temperature, and this is chosen to match either the mean emission-weighted temperature of the gas or to achieve the best fit to the projected gas temperature as a function of radius. In fact, there is generally a narrow range of this parameter which yields a reasonable temperature distribution for the cluster – one which does not increase rapidly to absurdly high values or one which does not fall rapidly to zero.

This is illustrated in Fig. 5 which is the radial dependence of projected gas temperature for the model fit to A2256 with $m_\nu = 1.9$ eV and $\chi = 0.32$. This is a non-cooling flow cluster with a low central density ($n_e = 0.0031$ cm $^{-3}$) and a large core radius ($r_c = 419$ kpc). The decline of surface brightness implies $\beta = 0.914$ (Reiprich & Böhringer 2002). Taking a central gas temperature of 7.5 keV produces a projected temperature profile (solid curve) which is consistent with that observed (De Grandi & Molendi 2002). The dashed curves illustrate the effect of taking a central gas temperature one-half keV higher or lower; it is evident that the resulting temperature profile is strongly dependent upon the initial assumed central gas temperature. This procedure has been carried out for several non-cooling flow clusters, and the characteristic temperature profile is

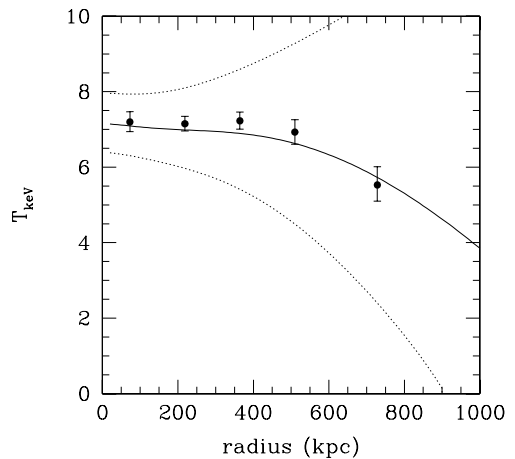


Figure 5. The predicted projected temperature distribution (emission weighted) in A2256 compared to that observed (de Grandi & Molendi 2002). The assumed neutrino mass is 1.9 eV and the degeneracy factor is $\chi = 0.32$. The β model parameters are $\beta = 0.914$, $r_s = 419$ hpc, $n_e = 0.0031$ and the mean emission-weighted gas temperature is 7.2 keV. The neutrinos are assumed to have a constant velocity dispersion appropriate to this temperature. The solid curve shows the prediction when the central gas temperature is taken to be 7.5 keV and the dashed curves show the distributions when the central temperature is taken to be 8 keV (upper curve) or 7 keV (lower curve).

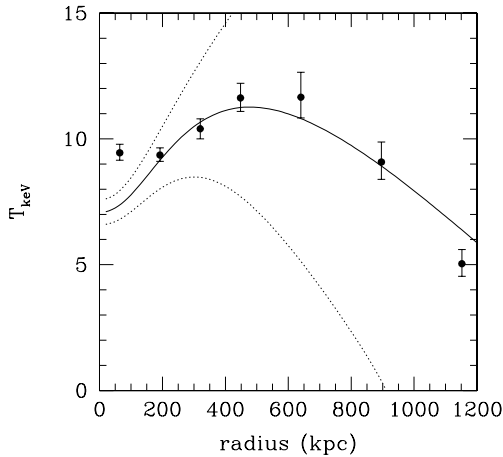


Figure 6. The predicted projected temperature distribution in A754 (solid curve) compared to that observed (de Grandi & Molendi 2002). The assumed neutrino mass is 1.9 eV and the degeneracy factor is $\chi = 0.32$. The β model parameters are $\beta = 0.698$, $r_c = 171$ kpc, $n_e = 0.00521 \text{ cm}^{-3}$. The central gas temperature is set to 6.44 keV. The two dotted curves are the predicted temperature distributions when the central gas temperature is set to 6.74 keV (upper curve) and 6.14 keV. This illustrates the sensitivity of the predicted distribution to central gas temperature in cooling flow clusters (high central electron density, small core radius).

similar to that shown here; that is, it is consistent with the observed profiles for non-cooling flow clusters.

For cooling flow clusters, the results are even more sensitively dependent upon the assumed central temperature, but, in general, the calculated temperature profiles agree with those observed for cooling flow clusters. This is shown in Fig. 6 which illustrates the predicted temperature profile (again with $m_\nu = 1.9$ eV and $\chi = 0.32$) for the cooling flow cluster A754 with β model parameters of $\beta = 0.698$, $r_c = 171$ kpc, $n_e = 0.00521 \text{ cm}^{-3}$ (Reiprich & Böhringer 2002). This is compared to the observed temperature profile by De Grandi & Molendi (2002). The dotted curves show the effect of increasing or decreasing the assumed central gas temperature by 0.3 keV; that is, the results here are extremely dependent upon this parameter.

For clusters with β model fits characterized by a small core radius and high central electron density, this is the general pattern predicted by the two-fluid models: a rapid rise in temperature followed by a gradual decline. In general, the predicted central temperature is lower than observed; the detailed agreement is less impressive than for non-cooling flow clusters and deteriorates when the neutrino mass is assumed to be smaller (1.8 eV is the lower limit imposed by this consideration). It should be kept in mind, however, that no baryonic component other than the gas is included in these calculations; the galaxies, and in particular, large central cD galaxies are not part of the mass modelling.

Overall, it appears that when the β model is imposed upon the gas density distribution, the temperature distribution required by self-consistency is in general agreement with the observed temperature profiles.

7 CAN INDIVIDUAL GALAXIES HAVE NEUTRINO DARK MATTER HALOES?

It would seem possible that individual galaxies could possess extensive neutrino dark haloes, even if the neutrino mass is as low as 2 eV. It is true that no primordial fluctuations on the scale of galaxies

would survive in the cosmic neutrino fluid due to free streaming. However, in the context of MOND, the baryonic component would collapse first and may well act as a seed for subsequent neutrino infall.

The neutrino halo would have a mass almost three times larger than the baryonic galaxy, but because of the phase-space constraint, it would be very extensive. Rewriting equation (17) in terms of rotation velocity, V_r , we would find

$$R_c^* = 1.3 \left(\frac{m_\nu}{1 \text{ eV}} \right)^{-4/3} \left(\frac{V_r}{200 \text{ km s}^{-1}} \right)^{1/3} \text{ Mpc} \quad (25)$$

for $m_\nu = 2$ eV the halo would extend to 500 kpc for a typical massive galaxy. The total mass of the neutrino halo on galactic scale (10–20 kpc) would be less than 1 per cent of the galaxy’s baryonic mass and the contribution of the halo to the observed rotation curve in the outer regions would be less than 10 km s^{-1} . Therefore, such a halo could have no effect on observed galaxy kinematics.

It would, however, affect the weak-lensing properties of galaxies on large scale. On the scale of hundreds of kpc, the halo mass would exceed the mass of the galaxy, and dominate the lensing signal. If, for example, the halo were flattened, then, with sufficient statistics, this could be apparent from the pattern of background galaxy images (Hoekstra, Yee & Gladders 2004).

The same considerations would apply to small groups of galaxies, which would also be expected possess extensive neutrino haloes. But given the transition temperature of 3 keV (see equation 18 and subsequent discussion), we would expect the overall dynamics of such systems to be determined by MOND and not by conventional theory. This expectation is consistent with the analysis of Milgrom (1998) who finds normal stellar mass-to-light ratios in small groups when analysed in terms of MOND (i.e. no additional discrepancy).

8 CONCLUSIONS

Non-baryonic dark matter in the form of active neutrinos certainly exists; only its contribution to the total mean cosmic density is in question. If the mass of the three neutrino types is as large as 2 eV, then neutrinos will be a constituent of massive rich clusters of galaxies. For this mass scale, the cosmological density ratio is of the order of the remaining mass discrepancy in clusters calculated via MOND; that is, such neutrinos would complete the mass budget of clusters in the context of MOND. Maintaining the cosmological mass ratio in clusters leads naturally to a dependence of central electron density on temperature ($n_e \propto T^{3/2}$) – a dependence which is consistent with density inferred from β model fits to the X-ray intensity distribution in non-cooling flow clusters (Fig. 2). This gas density–temperature dependence corresponds precisely to the constant entropy floor that has been proposed to account for the steepness of the observed luminosity–temperature relation, but in this case no preheating of the cosmic baryon fluid is needed; the floor is provided by the phase-space constraints on neutrino density. Of course, in cooling flow clusters one would expect there to be an inward flow of gas and breaking of the cosmological density ratio – with resulting higher electron densities.

Assuming that the cosmic density ratio of the two fluids is maintained and that the neutrinos and baryons have the same velocity dispersion, then only the neutrino mass and the degeneracy factor need be specified in order to calculate hydrostatic two-fluid models of clusters. Such models, with $m_\nu \approx 2$ eV, exhibit the observed X-ray luminosity–temperature relation – including a break to a steeper relation for lower temperature clusters (Fig. 4). If the gas density distribution is constrained to follow the fitted β models for

clusters, then the temperature distribution of the gas required for self-consistency resembles that in actual clusters – a general decline over hundreds of kpc for non-cooling clusters, but a rapid rise followed by a decline for cooling flow clusters (Figs 5 and 6).

Thus these calculations support the suggestion that the missing component in clusters of galaxies, in the context of MOND, may be neutrinos with mass near the present experimental upper limit. A remaining observational problem with this is that some clusters – primarily cooling flow clusters – apparently require a higher central density of dark material than is permitted by the phase-space constraints on neutrinos (Sanders 2003). This is evident both from X-ray observations and observations of strong gravitational lensing in the centres of some clusters where the implied central mass within an Einstein ring radius (100–200 kpc) may be in excess of $10^{13} M_{\odot}$ (Sanders 1999). Of course, the presence of a massive central galaxy and its effect on the distribution of neutrinos has not been included here, but it may well be that an additional undetected baryonic component in cooling flow clusters – perhaps resulting from the cooling flow itself – is required. I have also not considered the possibility that there is at least one massive sterile neutrino – a possibility with some theoretical and experimental motivation (Bilenky et al. 2003). In any case, the greatest part of the discrepancy in clusters would be accounted for by the three types of active neutrinos if their mass is near 2 eV.

We will not have to wait long for this possibility to be falsified (or confirmed). Currently planned β decay experiments (Osipowicz et al. 2001) will push the upper limit on the electron neutrino mass to a few tenths of an electron volt within a few years. If it turns out to be the case that $m_{\nu} \approx 2$ eV, then, with MOND, the old problem of clusters is solved.

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REFERENCES

- Aguirre A., Schaye J., Quataert E., 2001, *ApJ*, 561, 520
 Angus G. W., Shan H. Y., Zhao H. S., Famaey B., 2007, *ApJ*, 654, L13
 Arnaud M., Evrard A. E., 1999, *MNRAS*, 305, 631
 Bekenstein J. D., 2004, *Phys. Rev. D*, 70, 083509
 Bilenky S. M., Giunti C., Grifols J. A., Massó E., 2003, *Phys. Rep.*, 379, 69
 Clowe D., Bradac M., Gonzalez A. H., Markevitch M., Randall S. W., Jones C., Zaritsky D., 2006, *ApJ*, 648, L109
 David L. P., Arnaud M., Forman W., Jones C., 1990, *ApJ*, 356, 32
 De Grandi S., Molendi S., 2002, *ApJ*, 567, 163
 Edge A., Stewart G. C., 1991, *MNRAS*, 252, 428
 Famaey B., Binney J., 2005, *MNRAS*, 363, 603
 Fields B., Sarkar S., 2006, in Yao Y. M. et al., eds, *Review of Particle Physics*, J. Phys., G33, 1
 Fukuda Y. et al., 1998, *Phys. Rev. Lett.*, 81, 1562
 Gerbal D., Durret F., Lachièze-Rey M., Lima-Neto G., 1992, *A&A*, 262, 395
 Groom D. E. et al., 2000, *Eur. Phys. J.*, C15, 1
 Hoekstra H., Yee H. C., Gladders M. D., 2004, *ApJ*, 606, 67
 Ikebe Y., Reiprich T. H., Böhringer H., Tanaka Y., Kitayama T., 2002, *A&A*, 383, 773
 Kofman L. K. A., Pogosyan D., Henry J. P., 1996, *ApJ*, 470, 102
 Kull A., Treumann R. A., Böhringer H., 1996, *ApJ*, 466, L1
 Kull A., Treumann R. A., Böhringer H., 1997, *ApJ*, 484, 58
 Landau L., Lifshitz E. M., 1980, *Statistical Physics, Part 1*, 3rd edn. Elsevier, Butterworth-Heinemann, Oxford
 Lloyd-Davies E. J., Ponman T. J., Cannon D. B., 2000, *MNRAS*, 315, 689
 Lynden-Bell D., 1967, *MNRAS*, 136, 101
 Madsen J., Epstein R. I., 1984, *ApJ*, 282, 11
 McGaugh S. S., 2004, *ApJ*, 611, 26
 Milgrom M., 1983, *ApJ*, 270, 365
 Milgrom M., 1984, *ApJ*, 287, 571
 Milgrom M., 1998, *ApJ*, 496, L89
 Milgrom M., Sanders R. H., 2007, *ApJ*, 658, L17
 Mohr J. J., Mathiesen B., Evrard A. E., 1999, *ApJ*, 517, 627
 Mohr J. J., Reese E. D., Ellingson E., Lewis A. D., Evrard A. E., 2000, *ApJ*, 544, 109
 Nakajima M., Morikawa M., 2007, *ApJ*, 655, 135
 Osipowicz A. et al. (KATRIN collaboration), 2001, preprint (hep-ex/0109033)
 Piffaretti R., Jetzer Ph., Kaastra J. S., Tamura T., 2005, *A&A*, 433, 101
 Ponman T. J., Cannon D. B., Navarro J. F., 1999, *Nat*, 397, 135
 Pontecouteau E., Silk J., 2005, *MNRAS*, 342, 901
 Pratt G. W., Böhringer H., Croston J. H., Arnaud M., Borgani S., Finoguenov A., Temple R. F., 2007, *A&A*, 461, 71
 Reiprich T. H., Böhringer H., 2002, *ApJ*, 567, 716
 Sanders R. H., 1994, *A&A*, 284, L31
 Sanders R. H., 1999, *ApJ*, 512, L23
 Sanders R. H., 2003, *MNRAS*, 342, 901
 Sanders R. H., McGaugh S. S., 2002, *ARA&A*, 40, 263
 Sanders R. H., Noordermeer E., 2007, *MNRAS*, preprint (astro-ph/0703352)
 Skordis C., Mota D. F., Ferreira P. G., Boehm C., 2006, *Phys. Rev. Lett.*, 96, 1301
 Spergel D. et al., 2006, *ApJS*, 170, 377
 The L. S., White S. D. M., 1988, *AJ*, 95, 1642
 Tremaine S. C., Gunn J. E., 1979, *Phys. Rev. Lett.*, 42, 408
 Treumann R. A., Kull A., Böhringer H., 2000, *New J. Phys.*, 2, 11.1
 Vikhlinin A., Kravtsov A., Forman W., Jones C., Markevitch M., Murray S. S., Van Speybroeck L., 2006, *ApJ*, 640, 691
 Zhao H. S., Famaey B., 2006, *ApJ*, 638, L9

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